## Lesson 14

Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

## Suggested Lesson Structure

| $\square$ Fluency Practice | $(12$ minutes) |
| :--- | :--- |
| Concept Development | $(38$ minutes) |
| Student Debrief | $(10$ minutes) |
| Total Time | $(60$ minutes) |

## Fluency Practice (12 minutes)

- Multiply Fractions 5.NF. 4 (4 minutes)
- Find the Volume 5.MD.5c (5 minutes)
- Physiometry 4.G. 1 (3 minutes)


## Multiply Fractions (4 minutes)

Materials: (S) Personal white board
Note: This fluency activity reviews M4-Lessons 13-16.
T: (Write $\frac{1}{2} \times \frac{1}{4}$.) Say the multiplication number sentence with the answer.
S: $\quad \frac{1}{2} \times \frac{1}{4}=\frac{1}{8}$.
Continue the process with $\frac{1}{2} \times \frac{1}{5}$ and $\frac{1}{2} \times \frac{1}{9}$.
T: (Write $\frac{1}{2} \times \frac{1}{8}$.) On your personal white board, write the number sentence with the answer.
S: (Write $\frac{1}{2} \times \frac{1}{8}=\frac{1}{16}$.)
T: (Write $\frac{1}{2} \times \frac{5}{8}$.) Say the multiplication sentence with the answer.
S: $\frac{1}{2} \times \frac{5}{8}=\frac{5}{16}$.
Repeat the process with $\frac{1}{4} \times \frac{5}{5}, \frac{1}{3} \times \frac{9}{8}$, and $\frac{3}{4} \times \frac{1}{7}$.
T: (Write $\frac{3}{4} \times \frac{3}{5}$.) Write the multiplication sentence with the answer.
S: (Write $\frac{3}{4} \times \frac{3}{5}=\frac{9}{20}$.)

Continue the process with $\frac{2}{3} \times \frac{3}{8}$.
T: (Write $\frac{1}{4} \times \frac{5}{8}$.) Write the number sentence with the answer.
S: (Write $\frac{1}{4} \times \frac{5}{8}=\frac{5}{32}$.)
T: (Write $\frac{2}{3} \times \frac{3}{2}$.) Try this problem.
S: (Write $\frac{2}{3} \times \frac{3}{2}=\frac{6}{6}=1$.)

## Find the Volume ( 5 minutes)

Materials: (S) Personal white board
Note: This fluency activity reviews volume concepts and formulas.
T: (Project a prism 3 units $\times 2$ units $\times 7$ units.
Write V = $\qquad$ units $\times$ $\qquad$ units $\times$ $\qquad$ units.) Find the volume.
S: (Write 3 units $\times 2$ units $\times 7$ units $=42$ units $^{3}$.)
T: How many layers of 6 cubes are in the prism?
S: 7 layers.
T: Write a multiplication sentence to find the volume starting with the
number of layers.
S: (Write $7 \times 6$ units $^{3}=42$ units $^{3}$.)
T : How many layers of 21 cubes are there?
S: 2 layers.
T : Write a multiplication sentence to find the volume starting with the number of layers.
S: $\quad\left(\right.$ Write $2 \times 21$ units $^{3}=42$ units $^{3}$.)
T : How many layers of 14 cubes are there?


S: 3 layers.
T : Write a multiplication sentence to find the volume starting with the number of layers.
S: $\quad\left(\right.$ Write $3 \times 14$ units $^{3}=42$ units $^{3}$.)
Repeat the process for the other prisms.

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## Physiometry (3 minutes)

Materials: (S) Personal white board
Note: Kinesthetic memory is strong memory. This fluency activity prepares students for Lesson 16.
T: Stand up.
S: (Stand up.)
T: (Point at side wall.) Point to the wall that runs parallel to the one I'm pointing to.
S : (Point to the opposite wall.)
T: (Point to back wall.)
S: (Point to front wall.)
T: (Point to side wall.)
S : (Point to the opposite side wall.)
T: (Point at front wall.)
S: (Point at back wall.)
T: (Stretch one arm up, directly at the ceiling. Stretch the other arm directly toward a wall, parallel to the floor.) What type of angle do you think I'm modeling with my arms?
S: A right angle.
T: Model a right angle with your arms.
S: (Stretch one arm up, directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.)
T: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.) Model another right angle.
S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)

## Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The Problem Set has been incorporated into the Concept Development. The problems in today's lesson can be time intensive. It may be that only two or three problems can be solved in the time allowed. Students will approach representing these problems from many perspectives. Allow students the flexibility to use the approach that makes the most sense to them.

## Suggested Delivery of Instruction for Solving Lesson 14's Word Problems

## 1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

## Date:

- Can you draw something? This may or may not be a tape diagram today. An area model may be more appropriate.
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

## 2. Calculate to solve and write a statement.

Give everyone two minutes to finish working on that question, sharing his work and thinking with a peer. All students should write their equations and statements of the answer.

## 3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

## Problem 1

George decided to paint a wall with two windows. Both windows are $3 \frac{1}{2} \mathrm{ft}$ by $4 \frac{1}{2} \mathrm{ft}$ rectangles. Find the area the paint needs to cover.
Students must keep track of three different areas to solve Problem 1. Using a part-whole tape diagram to represent these areas may be helpful to some students, while others may find using the area model to be more helpful. Students have choices in the strategy for computing the areas as well. Some may choose to use the distributive property. Others may choose to multiply improper fractions. Once students have solved, ask them to justify their choice of strategy. Were they able to tell which strategy to use from the beginning? Did they change direction once they began? If so, why? Flexibility in thinking about these types of problems should be a focus.



$$
103-31 \frac{1}{2}=71 \frac{1}{2}
$$

Areatobe painted is $71 \frac{1}{2} \mathrm{ft}^{2}$

$15 \frac{3}{4} \times 2=30^{\frac{6}{4}}$


Area of windows:
$3 \frac{1}{2} \times 4 \frac{1}{2}=12+2+1 \frac{1}{2}+\frac{1}{4}=15 \frac{3}{4}$
$2 \times 15 \frac{3}{4}=30 \frac{6}{4}=31 \frac{1}{2} \mathrm{ft}^{2}$
Area of wall:
$8 \times 12 \frac{7}{8}=96+7=103 \mathrm{ft}^{2}$
Area to paint:
$103-31 \frac{1}{2}=71 \frac{1}{2} \mathrm{ft}^{2}$ The paint needs to cover
$71 \frac{1}{2}$ square feet.

## Problem 2

Joe uses square tiles, some of which he cuts in half, to make the figure pictured to the right. If each square tile has a side length of $2 \frac{1}{2}$ inches, what is the total area of the figure?

The presence of the triangles in the design may prove challenging for some students. Students who understand area as a procedure of multiplying sidesbut do not understand the meaning of area-may need scaffolding to help them reason about mentally reassembling the 6 halves to find 3 whole tiles.


$$
\begin{aligned}
& 10 \text { whole tiles }+6 \text { half tiles }=13 \text { +ils } \\
& \text { Area of a tile: } 2 \frac{1}{2} \text { in } 2 \frac{1}{2} \text { in } \\
& 13 \times 6 \frac{1}{4} \mathrm{in}^{2}=81 \frac{1}{4} \mathrm{in}^{2} \\
& \begin{array}{r}
13 \\
\times 6 \\
\hline
\end{array} \frac{13}{4}=3 \frac{1}{4} \\
& \text { The total area is } 81 \frac{1}{4} \text { square inches. } \\
& \frac{5}{2} \times \frac{5}{2}=\frac{25}{4}=6 \frac{1}{4} \\
& \text { Alleles: } 6^{\frac{1}{4}} \times 13=(6 \times 13)+\left(\frac{1}{4} \times 13\right) \\
& =78+\frac{13}{4} \\
& =78+3 \frac{1}{4} \\
& =81 \frac{1}{4}
\end{aligned}
$$

## NOTES ON <br> MULTIPLE MEANS OF ACTION AND EXPRESSION:

If students struggle with this problem, give them 13 square units and allow them to make designs with the tiles and find the areas. They will quickly see that the layout of the tiles does not change the area the tiles cover. They can then recreate the design in Problem 2, physically reassembling the half tiles as necessary to reason about the wholes.

## Problem 3

All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three?


## NOTES ON <br> MULTIPLE MEANS <br> OF ENGAGEMENT:

Problem 3 might be extended by inviting students to research actual carpet prices from local ads or the Internet and calculate what such a project might cost in real life. Comparison between the costs of using different types of flooring (hardwood versus carpet, for example) may also be made.

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While this problem is a fairly straightforward, additive area problem, an added complexity occurs at finding the dimensions of Room C. The complexity of this problem also lies in the need to keep three different areas organized before finding the total area. Again, once students have had an opportunity to work through the protocol, discuss the pros and cons of various approaches, including the reasoning for their choice of strategy.

$$
=378 \frac{6}{8}+12 \frac{5}{2}=351 \frac{1}{2}
$$



Total: $391 \frac{3}{8}$
$351 \frac{4}{8}$
$\frac{+351 \frac{6}{8}}{1093 \frac{13}{8}}$
$336+15^{\frac{3}{4}}$

$$
=351^{\frac{3}{4}}
$$

$$
=1,094 \frac{5}{8} .
$$

All.in-One needs $1,094 \frac{5}{8} \mathrm{ft}^{2}$

Room C: $21 \times 16 \frac{3}{4}=336+15 \frac{3}{4}$

$$
25 \frac{1}{4}-4 \mathrm{~m}^{\prime}=21 \quad=351 \frac{3}{4} \mathrm{ft}^{2}
$$

$$
\begin{array}{r}
21 \\
\times 16 \\
\hline 126
\end{array} \quad \frac{63}{4}=15 \frac{3}{4}
$$

$1,094 \frac{5}{8} \mathrm{ft}^{2}$ of carpet are needed.
$\frac{210}{336}$
Room A: $25 \frac{1}{1} \times 15 \frac{1}{2}=375+12 \frac{1}{2}+3 \frac{3}{4}+\frac{1}{8}$
Room A: $25 \frac{1}{1} \times 15 \frac{1}{2}=375+12 \frac{1}{2}+3 \frac{3}{4}+\frac{1}{8}$
$\begin{aligned} 25 & =390+\frac{11}{8} \\ \times 15 & =391 \frac{8}{8} 8\end{aligned}$
$\begin{aligned} 25 & =390+\frac{11}{8} \\ \times 15 & =391 \frac{8}{8} 8\end{aligned}$
Rcom B: $18 \frac{1}{2} \times 19=342+9 \frac{1}{2}$
Rcom B: $18 \frac{1}{2} \times 19=342+9 \frac{1}{2}$
$=391+351+351+\frac{3}{8}+\frac{1}{2}+\frac{3}{4}$
$=391+351+351+\frac{3}{8}+\frac{1}{2}+\frac{3}{4}$
$\times 18=351 \frac{1}{2} \mathrm{ft}^{2}$
$\times 18=351 \frac{1}{2} \mathrm{ft}^{2}$
$=1,093+\frac{13}{8}$
$=1,093+\frac{13}{8}$
$=1,094 \frac{5}{8} \mathrm{ft}^{2}$
$=1,094 \frac{5}{8} \mathrm{ft}^{2}$
$=390+\frac{11}{8}$
$=390+\frac{11}{8}$
$=391 \frac{3}{8}$
$=391 \frac{3}{8}$

## Problem 4

Mr. Johnson needs to buy sod for his front lawn.
a. If the lawn measures $36 \frac{2}{3} \mathrm{ft}$ by $45 \frac{1}{6} \mathrm{ft}$, how many square feet of sod will he need?
b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

The dimensions of the yard are larger than any others in the Problem Set to encourage students to use the distributive property to find the total area. Because the total area ( $1,656 \frac{1}{9} \mathrm{ft}^{2}$ ) is numerically closer to 1,656 , students may be tempted to round down. Reasoning about the $\frac{1}{9} \mathrm{ft}^{2}$ area can provide an opportunity to discuss the pros and cons of sodding that last fraction of a square foot. In the final component of the protocol, ask the following or similar questions:

- Is it worth the extra money for such a small amount of area left to cover? While 19 cents is a small cost, what if the sod had been more expensive?
- What if the costs had been structured so that the last whole square foot of sod had lowered the price of the entire amount?
- What could Mr. Johnson do with the other 8 ninths?

| Area | Price per square <br> foot |
| :--- | :---: |
| First $1,000 \mathrm{sq} \mathrm{ft}$ | $\$ 0.27$ |
| Next 500 sq ft | $\$ 0.22$ |
| Additional square feet | $\$ 0.19$ |

## Sod Prices

$$
\begin{aligned}
& \text { a. } 36 \frac{2}{3} \times 45 \frac{1}{6} \\
& \begin{array}{ll}
45 & 36 \times \frac{1}{6}=6 \\
\times 36 & 45 \times \frac{2}{2}=30 \\
270 & \\
\frac{210}{270} & \frac{2}{3} \times \frac{1}{63}=\frac{1}{9} \\
\hline 1,620 & 1,620+30+6+\frac{1}{9}=1,656 \frac{1}{9}
\end{array} \\
& \text { He sets } 1,656 \frac{1}{9} \mathrm{ft}^{2} \text { of sod. } \\
& \text { b. } 1.657 \text { whole spf.: } \\
& 1000 \times \$ 0.27=\$ 270.00 \\
& 500 \times \$ 0.22=\$ 110.00 \\
& 157 \times \$ 0.19=\frac{\$ .29 .83}{\$ 409.83} \\
& \$ 409.83 \\
& \text { He will have to pay } \\
& \$ 409.83 \text {. }
\end{aligned}
$$



Mr Johnson needs to buy. 1657 sq. ft of sod and it will cost $\$ 409.83$.

## Problem 5

Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ of an inch.
a. Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.
b. Find the area of the quilt.

There are many ways to lay out the quilt squares. Allow students to draw their layout, and then compare the perimeters. Ask the following questions:

- Does the difference in perimeter affect the area? Why or why not?
- Are there advantages to one arrangement of the blocks over another? (For example, lowering the cost for an edging by minimizing the perimeter or fitting the dimensions of the quilt to a specific wall or bed size.)

Problem 5 hearkens back to Problem 2 but with an added layer of complexity. Students might be asked to compare and contrast the two problems. In this problem, students must account for the seam allowances on all four sides of the quilt squares before finding the area. Students find that each quilt block becomes 42 $\frac{1}{4}$ inches square after sewing and may simply multiply this area by 24 .

## NOTES ON

MULTIPLE MEANS
OF ENGAGEMENT:
This problem may be extended for students who finish early. Ask them to find the arrangement that gives the largest perimeter, then the smallest. The problem can also be changed to having seams only between squares so there are three different square areas to calculate. Another extension could be offered by asking students to find the area of the seams. (Find the unfinished area of the 24 squares, and subtract the finished area.)


The quilt's area is $1,014 \mathrm{in}^{2}$.

## Student Debrief (10 minutes)

Lesson Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Do these problems remind you of any others that we've seen in this module? In what ways are they like other problems? In what ways are they different?

- What did you learn from looking at your classmates' drawings? Did that support your understanding of the problems in a deeper way? When you checked for reasonableness, what process was used?
- When finding the areas, which strategy did you use more often-distribution or improper fractions? Is there a pattern to when you used which? How did you decide? What advice would you give a student who wasn't sure what to do?
- Which problems did you find the most difficult? Which one was easiest for you? Why?


## Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


Name $\qquad$ Date $\qquad$

1. George decided to paint a wall with two windows. Both windows are $3 \frac{1}{2} \mathrm{ft}$ by $4 \frac{1}{2} \mathrm{ft}$ rectangles. Find the area the paint needs to cover.

$$
12 \frac{7}{8} \mathrm{ft}
$$


2. Joe uses square tiles, some of which he cuts in half, to make the figure below. If each square tile has a side length of $2 \frac{1}{2}$ inches, what is the total area of the figure?

3. All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three?

4. Mr. Johnson needs to buy sod for his front lawn.
a. If the lawn measures $36 \frac{2}{3} \mathrm{ft}$ by $45 \frac{1}{6} \mathrm{ft}$, how many square feet of sod will he need?
b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

## Sod Prices

| Area | Price per square <br> foot |
| :--- | :---: |
| First $1,000 \mathrm{sq} \mathrm{ft}$ | $\$ 0.27$ |
| Next 500 sq ft | $\$ 0.22$ |
| Additional square feet | $\$ 0.19$ |

5. Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ of an inch.
a. Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.
b. Find the area of the quilt.

Name $\qquad$ Date $\qquad$
Mr. Klimek made his wife a rectangular vegetable garden. The width is $5 \frac{3}{4} \mathrm{ft}$, and the length is $9 \frac{4}{5} \mathrm{ft}$. What is the area of the garden?

Name $\qquad$ Date $\qquad$

1. Mr. Albano wants to paint menus on the wall of his café in chalkboard paint. The gray area below shows where the rectangular menus will be. Each menu will measure 6 ft wide and $7 \frac{1}{2} \mathrm{ft}$ tall.


- How many square feet of menu space will Mr. Albano have?
- What is the area of wall space that is not covered by chalkboard paint?

2. Mr. Albano wants to put tiles in the shape of a dinosaur at the front entrance. He will need to cut some tiles in half to make the figure. If each square tile is $4 \frac{1}{4}$ inches on each side, what is the total area of the dinosaur?

3. A-Plus Glass is making windows for a new house that is being built. The box shows the list of sizes they must make.

15 windows $4 \frac{3}{4} \mathrm{ft}$ long and $3 \frac{3}{5} \mathrm{ft}$ wide
7 windows $2 \frac{4}{5} \mathrm{ft}$ wide and $6 \frac{1}{2} \mathrm{ft}$ long

How many square feet of glass will they need?
4. Mr. Johnson needs to buy seed for his backyard lawn.

- If the lawn measures $40 \frac{4}{5} \mathrm{ft}$ by $50 \frac{7}{8} \mathrm{ft}$, how many square feet of seed will he need?
- One bag of seed will cover 500 square feet if he sets his seed spreader to its lowest setting and 300 square feet if he sets the spreader to its highest setting. How many bags of seed will he need if he uses the highest setting? The lowest setting?

